Nuclear Matter Studies with Density-dependent Meson-Nucleon Coupling Constants

M. K. Banerjee[1,2] and J. A. Tjon[1,3]

[1]Department of Physics, University of Maryland, College Park, Maryland 20742, USA

[2]Instute for Kernphysik, Forschungszentrum Jülich, 52425 Jülich, Germany

[3]Institute for Theoretical Physics, University of Utrecht, 3508 TA Utrecht, the Netherlands

Due to the internal structure of the nucleon, we should expect, in general, that the effective meson nucleon parameters may change in nuclear medium. We study such changes by using a chiral confining model of the nucleon. We use density-dependent masses for all mesons except the pion. Within a Dirac-Brueckner analysis, based on the relativistic covariant structure of the NN amplitude, we show that the effect of such a density dependence in the NN interaction on the saturation properties of nuclear matter, while not large, is quite significant. Due to the density dependence of the $g_{\sigma NN}$, as predicted by the chiral confining model, we find, in particular, a looping behavior of the binding energy at saturation as a function of the saturation density. A simple model is described, which exhibits looping and which is shown to be mainly caused by the presence of a peak in the density dependence of the medium modified σN coupling constant at low density.

The effect of density dependence of the coupling constants and the meson masses tends to improve the results for E/A and density of nuclear matter at saturation. From the present study we see that the relationship between binding energy and saturation density may not be as universal as found in nonrelativistic studies and that more model dependence is exhibited once medium modifications of the basic nuclear interactions are considered.

PACS numbers: 21.65.+f, 14.20.-c

I. INTRODUCTION

Since a nucleon is not a point object, but has structure, it must undergo changes when placed inside a nucleus. Among other properties, the meson-nucleon coupling constants may change. If this happens, it should affect the NN force. These effects have to be small. Otherwise, traditional nuclear physics, where one uses free-space two-body force would have failed badly. But even small changes in the NN force may have noticeable effect on the properties of nuclear matter. The purpose of this paper is to investigate possible changes of meson nucleon coupling constants due to the quark structure of the nucleon and their ultimate effects on the saturation properties, such as the density ρ_0 and the binding energy $-E/A$ of nuclear matter.

The NN force in nuclear matter may become density dependent due to a wide variety of reasons. Any time one eliminates some degrees of freedom the resulting effective interaction becomes density dependent, the Brueckner G-matrix being the most widely known example. Another recent example is the work of Li *et al* [1] on the effective interaction to be used in a mean field calculation which reproduces the results of a Brueckner-Hartree-Fock calculation.

The density dependence studied here involves excitations of N* ($I = J = 1/2$ resonances) degrees of freedom.[1] Ultimately we are interested in the change of the NN force in the medium. In relativistic nuclear physics the most important forces are mediated by a scalar-isoscalar field σ with $m_\sigma \simeq 600$ MeV and a vector-isoscalar field ω with $m_\omega = 783$ MeV. Thus the force range is short and the neighboring nucleons which can alter the internal structure of the interacting nucleons must also be close. Due to the Exclusion principle

[1] The nuclear matter is an isoscalar and isotropic medium. It cannot change the spin or the isospin of a nucleon. It is also translationally invariant. But, as we will see later, it can still produce internal excitations in a nucleon.

and short-range correlations, the effective density of the polarizing nucleons, denoted as $\xi\rho$, is less than half the normal nuclear density. It is quite possible that the N^* excitations produced by the neighboring nucleons may be treated perturbatively. However, an earlier study of this problem [2] established that a fairly large number of resonances contribute. The mean field approach, which generates the eigen-combination of N and N^*s as the lowest state in the field due to the neighboring nucleons is a more expeditious way of calculating the effect.

Our quantitative studies are based on the following strategy. We describe the structure of the nucleon with a model called the chiral confining model (CCM) [3,4]. Specifically, we use the TOY [4] version of this model. The role of nuclear matter is simulated with baths of external σ and ω fields, the vacuum values being $\langle\sigma\rangle_{vac} = -F_\pi = -93$ MeV and $\langle\omega\rangle_{vac} = 0$, respectively. The nucleon structure problem is solved in the presence of these bath fields. Then various properties of the nucleon, including meson nucleon coupling constants, are calculated for ranges of values of the two bath fields.

The fact that the σ and the ω fields are, in turn, produced by the nucleons through field-dependent coupling constants allows us to obtain density dependences of the σ and the ω fields by solving appropriate nonlinear self-consistency equations. Once this has been done, we can use the known field dependences of various properties of the nucleon to obtain their density dependences.

We indicate the density-dependent values of various properties of the nucleon with a star. Thus $g_{\sigma NN}$ denotes the free space σ NN coupling constant and $g^*_{\sigma NN}$ the same quantity in nuclear matter. The latter is always dependent on ρ, the nuclear density. We may note that the properties of nuclear matter depend principally on $g^*_{\sigma NN}$, $g^*_{\omega NN}$, $g^*_{\pi NN}$, $g^*_{\rho NN}$ and $f^*_{\rho NN}$, where the last is the ρ Pauli coupling coefficient.

A necessary input is the field-dependence of the exchanged meson masses[2]. We use the

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[2]The possible importance of density dependence of meson masses was first stressed by Brown and

simple model that the meson masses are linearly dependent on the σ field, keeping the pion mass fixed. [3] The coefficient of linear dependence of σ and ω masses on the σ field is chosen to give $m^*/m = 0.92$ at normal nuclear density.

The changes, as found in the CCM, clearly will have effect on the properties of nuclear matter. To study this we adopt the relativistic many-body approach [6–8] . Specifically, we carry out a relativistic Dirac-Brueckner calculation of the properties of nuclear matter using the one-boson-exchange model of Ref. [9]. For the quasi-potential version of this model the full Dirac structure of the NN amplitude in free space has been constructed [10]. The resulting so-called IA2 representation can be used to determine the saturation properties of nuclear matter [8]. Modifying the free space T-matrix to also include Pauli-blocking and introducing our density-dependent meson-nucleon coupling constants, self-consistent relativistic Dirac-Brueckner calculations were performed in the manner of Ref. [8] for a range of $\xi\rho$ the effective density of the polarizing nucleons.

Our main results are that

(i) the effects of the density-dependent meson nucleon coupling constants, arising out of the quark structure of the nucleons, on the saturation density and $-E/A$ at saturation are small but not negligible,

(ii) and they do tend to improve the results.

The next section contain brief introduction to the Chiral Confining Model (CCM) and its TOY version. Section 3 describes the calculation of nucleon properties as functions of bath σ and ω fields. These results are used in the next section to extract density dependences of nucleon properties. Section 5 describes the relativistic treatment of nuclear matter using the

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his collaborators [5]

[3]Because of charge conjugation symmetry the masses of mesons, considered here, cannot depend linearly on ω . The dependence must be quadratic or of higher even power.

Dirac-Brueckner approach. The last section presents the main results, discussions of these results and concluding remarks.

II. THE CHIRAL CONFINING MODEL

An early attempt at extracting density dependence of nucleon properties using CCM is described in Ref. [2]. The present paper contains two significant improvements - inclusions of the instanton induced interaction and the pion cloud contributions to $g_{\sigma NN}$, $g_{\rho NN}$ and $f_{\rho NN}$ coupling constants. The CCM has been described in detail in [2–4]. Here we give a brief review.

The CCM is based on the notion of color dielectric function as introduced by Nielsen and Pàtkos [11]. By considering the average of all possible link operators, starting from $x-\epsilon$ and ending on x with the paths completely contained in a four-dimensional hypercube of side L, they introduced a color singlet, Lorentz scalar quantity $K(x)$ and a color octet, Lorentz vector, coarse grained gluon field B_μ^a:

$$K(x) = \lim_{\epsilon\to 0}\frac{1}{N_c}Tr \quad \left[\langle e^{-i\int_{x-\epsilon}^{x} dy\cdot A(y)}\rangle\right],$$
$$B_\mu = \frac{1}{2}\sum_a \lambda_a B_\mu^a = \lim_{\epsilon\to 0} i\frac{\partial}{\partial\epsilon} \quad \left[\langle e^{-i\int_{x-\epsilon}^{x} dy\cdot A(y)}\rangle\right].$$

Upon integrating out the QCD gluon fields in favor of these new collective variables one obtains the Nielsen-Pàtkos lagrangian in the form of a derivative expansion:

$$\mathcal{L}_{NP} = \bar\psi(x)[iK(x)\frac{1}{2}\overleftrightarrow{\not\partial} - K(x)m_q - g\not B(x)]\psi(x) - \frac{K^4}{4}G^a_{\mu\nu}G^{a\,\mu\nu} + \ldots . \tag{1}$$

The gauge field is B_μ^a/K and not B_μ^a. The coarse grained field tensor is

$$G^a_{\mu\nu} = \partial_\mu\frac{B^a_\nu}{K} - \partial_\nu\frac{B^a_\mu}{K} + f^{abc}\frac{B^b_\mu}{K}\frac{B^c_\nu}{K}. \tag{2}$$

From the gluonic term one identifies $\epsilon = K^4$ as the color dielectric function. Nielsen and Pàtkos conjectured that

$$\langle K\rangle_{vac} = 0. \tag{3}$$

This conjecture, crucial for our model, has been justified from the lattice gauge point of view by Lee *et al* [12].

A quark has ever-present interaction with the quark condensate of the vacuum. If $\langle K\rangle_{vac} = 0$, the interaction will appear to be infinitely strong compared to quark kinetic energy and a quark cannot exist in that region. It can only reside in the region where $\langle K\rangle_{vac} \neq 0$. A color singlet quark system can polarize the vacuum and change the value of K away from zero, thus dynamically generating the *bag* where the quarks can stay.

The Nielsen-Pàtkos lagrangian recognizes the existence of gluon condensate through the vanishing of $\langle K\rangle_{vac}$. However, the quark condensate is not manifest. Without it one cannot develop an effective lagrangian which contains the physics of the interaction of a quark with the quark condensate. We deal with this problem by conjecturing that one can integrate out the coarse grained gluon fields in favor of meson fields as new collective variables [4,13,14].

It is also necessary to introduce, following Nielsen and Pàtkos [11], a new color singlet, Lorentz scalar field, χ, proportional to K by the equation

$$K(x) = g_\chi \chi(x). \tag{4}$$

Being related to K, a purely gluonic object, the χ field is a member of the glueball family. Hence, it is a chiral singlet, a fact evident from the first term of the Nielsen-Pàtkos lagrangian given by Eq. (1). Large N_{color} analysis shows that it is a hybrid field [4,15]. The result (3) imposes the requirement that

$$\langle\chi\rangle_{vac} = 0. \tag{5}$$

Retaining minimum powers of fields and their derivatives the basic lagrangian of the CCM has the form:

$$\begin{aligned}\mathcal{L}_{CCM} &= K(x)\bar{\psi}(x)[i\frac{1}{2}\overleftrightarrow{\partial\!\!\!/} - m_q]\psi(x)\\ &+ \bar{\psi}(x)\frac{g_\pi\{\sigma(x) + i\gamma_5\vec{\tau}\cdot\vec{\pi}\} + g_\omega\omega\!\!\!/ + g_\rho\vec{\tau}\cdot(\vec{\rho\!\!\!/} + \gamma_5\vec{A\!\!\!/}_1)}{K(x)}\psi(x)\\ &+ \mathcal{L}_{meson} + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi).\end{aligned} \tag{6}$$

The quantities g_π, g_ω and g_ρ are the quark-meson coupling constants. The quantity m_q is the current quark mass. Its value is set at 7.5 MeV. It contributes 17 MeV to the πN sigma term [4]. Because of its negligible role in the present work we will neither refer to this term nor count it as a parameter in our subsequent discussions. However, it is included in the actual numerical work.

We use the chiral invariant lagrangian of Lee and Nieh [16] for $\mathcal{L}_{meson}$. The lagrangian ensures that $\langle\sigma\rangle_{vac} = -F_\pi$ and that the mesons have their respective physical masses when calculated at the classical (or tree) level. The fields π and σ form a $(1/2\times 1/2)$ representation of $SU_L(2) \times SU_R(2)$, while the fields $\vec{\rho} \pm \vec{A}_1$ form $(1,0)$ and $(0,1)$ representations.

To complete the definition of $\mathcal{L}_{CCM}$ one must specify the χ potential. Since nothing substantial is known about it, we try two simple forms:

$$\begin{aligned} \mathrm{PURE\,MASS}: U(\chi) &= \frac{1}{2}m_\chi \chi^2, \\ \mathrm{QUARTIC}: U(\chi) &= \frac{1}{2}m_\chi \chi^2 (1-\chi/\chi_0)^2. \end{aligned} \tag{7}$$

The QUARTIC potential has two minima, the one at $\chi = 0$ describes the vacuum while the other at $\chi = \chi_0$ is an 'excited' state, which for simplicity we keep degenerate with the vacuum. The value of χ_0, the location of the second minimum is set at 40 MeV. The hybrid mass, m_χ is set at 1400 MeV. As we will discuss later, the results of the calculations depend largely on one particular combination of these parameters and rather weakly on individual ones.

Any mean field calculation containing isovector fields requires using states which do not have good isospin symmetry. To accomplish this we use hedgehog spinors and fields, introduced first for π and σ fields by Chodos and Thorn [17] and extended to vector meson fields by Broniowski *et al* [18]. We closely follow the mean field analysis of Ren *et al* [19].

It should be emphasized that the appearence of K in the denominator of the quark-meson interaction term is not *ad hoc*. It is obtained by matching the K dependence of the four-quark interactions which arise on the one hand from $\mathcal{L}_{NP}$ by integrating out the B_μ fields and on the other hand from $\mathcal{L}_{CCM}$ by integrating out the meson fields [4,20,21].

The presence of the factor K with $\bar{\psi} i \frac{1}{2} \overleftrightarrow{\not\partial} \psi$ changes the field canonically conjugate to ψ from the usual $i\psi^\dagger$ to $iK\psi^\dagger$. As a result the quark term in all Noether's currents carries the factor K. The transformation $\sqrt{K}\psi(x) \rightarrow \psi(x)$ makes the pair ψ and $i\psi^\dagger$ canonical and removes the factor K from all Noether's currents. Two changes occur in $\mathcal{L}_{CCM}$. The free quark term no longer has the factor K, while the quark meson interaction acquires the factor K^2 in place of K in the denominator.

Mean field treatment of $\mathcal{L}_{CCM}$ with a variety of reasonable sets of parameters reveals [4,19,21] that even where the quark density is large the meson fields differ only slightly from their respective vacuum values. This suggests strongly that we introduce a simplified version of CCM. The simplified version, which we call the TOY model, consists of fixing all meson fields at their vacuum values:

$$
\begin{aligned}
\langle\sigma\rangle_{vac} &= -F_\pi, \\
\langle\vec{\pi}\rangle_{vac} = \langle\omega\rangle_{vac} = \langle\vec{\rho}\rangle_{vac} = \langle\vec{A}_1\rangle_{vac} &= 0,
\end{aligned} \tag{8}
$$

leaving only the quark fields and the χ field as dynamical variables. The Toy model lagrangian, in terms of the canonical quark field, is given below.

$$
\mathcal{L}_{Toy} = \bar{\psi}(x)[i\frac{1}{2} \overleftrightarrow{\not\partial} - m_q - \frac{g_\pi F_\pi}{(g_\chi \chi)^2}]\psi(x) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi). \tag{9}
$$

Not counting m_q, the TOY model has only two parameters, $g_\pi F_\pi / g_\chi^2$ and m_χ in the PURE MASS version and one additional parameter, χ_0, in the QUARTIC version. Formation of the bag is even more transparent in the TOY model as the constituent quark mass term, $\bar{\psi}\frac{g_\pi F_\pi}{(g_\chi\chi)^2}\psi$, becomes infinite when $\chi \rightarrow 0$.

The unusual form of the quark-meson interaction with the factor of $K^2 = (g_\chi\chi)^2$ in the denominator is of some importance in the present investigation. In its absence a constant external ω field will merely shift the energy of the quark without exciting it to higher states. Thus the nucleon will not be polarized. A constant σ field will excite the quark. Orthonormality of different eigenstates of the Dirac hamiltonian ensures that $\int d^3 r u'^\dagger u = 0$, but not the vanishing of $\int d^3 r \bar{u}' u$. The latter gives the effect of a constant external σ field.

The presence of the factor $(g_\chi \chi)^2$ enables a constant external ω field to excite the quark and enhances the ability of a constant external σ field to do the same.

Because of its importance it is necessary to provide some evidence for the presence of $(g_\chi \chi)^2$ in the denominator of the quark-meson interaction term. Fortunately, there is a verifiable consequence of this feature. Gauge invariance ensures that there is no such modification of the photon-quark interaction. Thus contrary to the conjectures of universal coupling [22] or current-field identity [23] the isovector couplings of the photon and the ρ meson differ by this K^{-2} factor. Because of this difference, CCM predicts that the ρ-nucleon Pauli coupling constant, $f_{\rho NN}$ should differ from the isovector anomalous magnetic moment, $\kappa_{I=1}$. The predicted [19] value is $f_{\rho NN}/\kappa_{I=1} = 1.4$, instead of 1 predicted by the two conjectures. Based on dispersion theoretic analysis of πN scattering data Höhler and Pietarinen [24] had estimated the ratio to be ~ 1.75. Our value is within the possible uncertainties of this result [25].[4]

The role of the pion field on baryon properties is never negligible. In the TOY model one takes account of it perturbatively [3,26] wherever appropriate. In the context of the present study such corrections are important in the calculations of the coupling constants of mesons of even G-parity, $g_{\sigma NN}$, $g_{\rho NN}$ and $f_{\rho NN}/2M$.

The Toy model with perturbative pionic correction gives poor results for the masses of N and Δ and for the nucleon charge radii [3,26]. McGovern *et al* [27] showed that there is approximate, but very good scaling behavior in the TOY model. If one chooses the parameters to get the nucleon mass right the values of most other quantities are nearly fixed. Thus the bad results for mass and size cannot be improved simultaneously by varying pa-

[4] In the MIT bag model also the photon and the ρ meson couple to the quark in different manner. Brown and Rho had exploited this feature to explain the value of $f_{\rho NN}/\kappa_{I=1}$. Dialing the chiral angle they fitted the value 1.75. Our result is approximately independent of parameters because of the scaling property mentioned later.

rameters. One needs a new interaction which is short-ranged and more attractive between singlet-singlet quark pairs than between triplet-triplet pairs. The magnetic one-gluon exchange interaction [28] fits the requirement exactly. However, the physics of exchange of a color octet tower of gluons is already incorporated in meson exchanges. We cannot include it again.

We solve the problem by including the instantanton induced 't Hooft interaction [29,30]. In the two flavor case of u and d quarks, the 't Hooft interaction generates interactions between quark pairs in flavor antisymmetric state only and the interaction is attractive and zero-ranged. So it fits our requirement without duplicating any mechanism already included. Using Shuryak's [31] description of the vacuum as an instanton liquid the strength of the 't Hooft interaction can be estimated in QCD, but not in CCM [3]. So we have to treat it as a free parameter. We fix it by fitting the $N - \Delta$ mass splitting with the 't Hooft interaction together with one-pion exchange contribution [3].

The method of calculation with the TOY model in the absence of external σ and ω has been described in detail in Ref. [3] and will not be repeated here. The changes due to the presence of external fields are straightforward [2]. Going back to $\mathcal{L}_{CCM}$, described by Eq. (6), we interpret σ and ω_0 as the bath fields. The vacuum expectation values of all other fields, $\vec{\pi}$, ω_i $(i = 1, 2, 3)$, $\vec{\rho}_\mu$ and $\vec{A}_{1\mu}$ are zero because nuclear matter has good isospin and good parity. The quark field and the χ field continue to be the only dynamical variables in the extended TOY model lagrangian. The new energy functional of a nucleon in a σ -ω bath is given below.

$$\begin{aligned} E = N_c \int d^3 r u^\dagger(\vec{r}) \left[-i\vec{\alpha} \cdot \vec{\nabla} - \frac{g_\pi \gamma_0 \sigma - g_\omega \omega}{(g_\chi \chi(\vec{r}))^2} \right] u(\vec{r}) \\ + \int d^3 r \left[\frac{1}{2} (\vec{\nabla} \chi(\vec{r}))^2 + U(\chi(\vec{r})) \right], \\ - 2C_s \int d^3 r \mathcal{Z}(r) (G^2(r) + F^2(r)). \end{aligned} \tag{10}$$

where the valence spinor, $u(\vec{r})$ is

$$u(\vec{r}) = \begin{pmatrix} G(r) \\ i\vec{\sigma}\cdot\hat{r}F(r) \end{pmatrix}\zeta, \tag{11}$$

and ζ is a 2-component Pauli spinor.

The last line in Eq. (10) represents the contribution of the instanton induced 't Hooft interaction with a regulating function, $\mathcal{Z}(r)$, which cuts off the interaction at a distance scale of 0.25 fm. The details may be found in Refs. [3,26].

Note that the external fields are constant in space and time. The vacuum value of the σ field is $-F_\pi$. Nuclear matter generates σ field with positive sign thus reducing the magnitude of the net σ . It is customary to represent the effect as a modification of the pion decay constant:

$$\sigma = -F_\pi + \sigma_{nmatter} = -F_\pi^*. \tag{12}$$

The ω in Eq. (10) is the time component of vector ω field. The static nuclear matter distribution can generate only the time component. However, a nucleon moving with velocity $\vec{v}$ in nuclear matter does see a space component $\vec{\omega} = -\vec{v}\omega/\sqrt{1-v^2}$. We ignore the role of this term.

III. FIELD DEPENDENCE OF NUCLEON PROPERTIES

We pick values of F_π^* in the range 63 MeV to 93 MeV in steps of 6 MeV and values of ω in the range 0 to 40 MeV in steps of 8 MeV. For each pair of F_π^* and ω we find the spinor and the χ field which will make the energy functional stationary. The condition of stationarity yields coupled nonlinear equations. Ref. [3] describes the method of solving these equations. Once the solutions are obtained we can calculate the desired properties of the nucleon in the σ -ω field bath.

We introduce the dimensionless variables x and y to describe, in units of m_π, the bath fields σ and ω :

$$\sigma - \langle\sigma\rangle_{vac} = F_\pi - F_\pi^* = x m_\pi. \tag{13}$$

$$\omega = y m_\pi, \tag{14}$$

We fit the results of every physical quantity Q of interest with the quadratic form:

$$Q(x,y) = a[1 + bx + cy + dx^2 + fxy + gy^2] = aF^Q(x,y). \tag{15}$$

TABLE I. Values of quantities a, b, c, d, f and g for the PUREMASS case. The quantities under the column a are the free space values.

Quantity	a	b	c	d	f	g
$(g_{\pi NN}/2M)^*$	0.83	1.72	-0.78	3.51	-5.71	0.13
$g^*_{\omega NN}$	11.02	1.10	-1.60	2.44	-2.90	3.22
$g^{(q)*}_{\sigma NN}$	2.34	3.14	-5.25	3.34	-21.77	6.99

TABLE II. Values of quantities a, b, c, d, f and g for the QUARTIC case. The quantities under the column a are the free space values.

Quantity	a	b	c	d	f	g
$(g_{\pi NN}/2M)^*$	0.92	1.62	-0.49	3.55	-3.36	0.045
$g^*_{\omega NN}$	13.72	0.74	-0.93	1.23	-1.18	1.45
$g^{(q)*}_{\sigma NN}$	4.92	1.67	-2.13	-2.39	-6.65	1.72

The results of our fit are shown in Tables I and II for the PUREMASS and QUARTIC case respectively. The quantity a is the value of Q in vacuum and $F^Q(x,y)$ is the quadratic polynomial in square brackets. The three coupling constants of special interest are $g^{(q)*}_{\sigma NN}$, $g^*_{\omega NN}$ and $(g_{\pi NN}/2M)^*$. The σ-nucleon coupling constant carries a superscript (q) to indicate that only the direct coupling of the σ field to valence quarks is included in $g^{(q)*}_{\sigma NN}$. The full coupling constant also includes coupling to the pion field produced by the valence quarks. The ω and the π fields have odd G-parity. Hence their source currents couple to 3 or higher odd powers of the π field. Since the π field is weak inside the nucleon, the higher power contributions are not important and have been omitted in the present calculation. The superscript (q) is redundant for $g^*_{\omega NN}$ and $(g_{\pi NN}/2M)^*$ and have been omitted.

The parts of the meson source current which show the coupling to quarks may be read off Eq. (6) describing $\mathcal{L}_{CCM}$. We list them below:

$$j^{(q)}_{\sigma} = g_{\pi}\frac{1}{K(x)}, \tag{16}$$

$$j^{(q)}_{\omega} = g_{\omega}\frac{\gamma_0}{K(x)}, \tag{17}$$

$$j^{(q)}_{\pi,\alpha} = g_{\pi}\frac{i\gamma_5\tau_\alpha}{K(x)}, \tag{18}$$

$$j^{(q,\mu)}_{\rho,\alpha} = g_{\rho}\frac{\gamma^\mu\tau_\alpha/2}{K(x)}. \tag{19}$$

The meson-nucleon coupling constants are defined in the usual way. The expressions listed below are for $\vec{p}\ \to 0$.

$$\langle N(\vec{(p)}) \mid j^{(q)}_{\sigma}(0) \mid N(\vec{(p)})\rangle = g^{(q)}_{\sigma NN}, \tag{20}$$

$$\langle N(\vec{(p)}) \mid j^{(q)}_{\omega}(0) \mid N(\vec{(p)})\rangle = g_{\omega NN}, \tag{21}$$

$$\langle proton(\vec{(p)}) \uparrow\mid j^{(q,\mu)}_{\rho,3} \mid proton(\vec{(p)}) \uparrow\rangle = \frac{1}{2}g^{(q)}_{\rho NN}, \tag{22}$$

$$\langle proton(\vec{(p)}) \uparrow\mid \frac{1}{2}\int d^3\vec{r}\ [\vec{r}\times\vec{j}^{(q)}_{\rho,3}]\ \mid proton(\vec{(p)}) \uparrow\rangle = \mu^{(q)}_{\rho NN}, \tag{23}$$

$$\langle proton(\vec{(p)}') \uparrow\mid j^{(q)}_{\pi,3}(0) \mid proton(\vec{(p)}) \uparrow\rangle = (\vec{p}' - \vec{p})_z\, g_{\pi NN}/2M. \tag{24}$$

The quantity $\mu_{\rho NN}$ appearing in Eq. (23) is the ρ-magnetic moment of the nucleon:

$$\mu_{\rho NN} = (g_{\rho NN} + f_{\rho NN})/2M. \tag{25}$$

We list below the results from CCM for the contributions to the various coupling constants coming from direct coupling of meson fields to the quarks.

$$g^{(q)*}_{\sigma NN} = g_\pi \int d^3r[G^2(r) - F^2(r)]/K^2(r), \tag{26}$$

$$(g_{\pi NN}/2M)^* = g_\pi \frac{10}{9} \int d^3r rG(r)F(r)/K^2(r), \tag{27}$$

$$g^*_{\omega NN} = 3g_\omega \int d^3r[G^2(r) + F^2(r)]/K^2(r), \tag{28}$$

$$g^{(q)*}_{\rho NN} = g_\rho \int d^3r[G^2(r) + F^2(r)]/K^2(r), \tag{29}$$

$$\mu^{(q)*}_{\rho NN} = g_\rho \frac{10}{9} \int d^3r rG(r)F(r)/K^2(r). \tag{30}$$

To obtain density-dependent, or equivalently field-dependent, coupling constants one must use K and spinor components G and F obtained in the presence of the bath fields. The stars should be removed for free space values of the coupling constants obtained with K, G and F appropriate for free space.

The nucleon mass term which appears as a divisor for $g_{\pi NN}$ and for the ρ magnetic coupling stands for the experimental value of the mass, *i.e.,* 939 MeV. It is used to define dimensionless coupling constants from the integrals of dimensions of length which appear in Eqs. (27) and (30). Our studies measures the density dependence of the quantities represented by the integrals.

A model of hadrons cannot anticipate what effective hadronic lagrangian its results will be used for. Hence, to remove possible misunderstanding we have stated in Eq. (24) the precise definition of the quantity $g_{\pi NN}/2M$ as it appears in our work. The expression on the left hand side involves the pion source current, usually well-defined in most quark based model of the nucleon. It is also well-defined in any effective hadronic lagrangian. If the effective lagrangian uses pseudoscalar πN coupling the quantity $g_{\pi NN}$ is the dimensionless coupling constant of the theory. If, on the other hand, it uses derivative coupling then $g_{\pi NN}/2M$ is the coupling constant of the theory, having the dimensions of length. Note, again, that M in the denominator of $g_{\pi NN}/2M$ for the derivative coupling theory is a fixed number by definition, usually 939 MeV.

The mesons with even G-parity, σ and ρ, also couple to the pion cloud, the contributions from which depend essentially quadratically on $(g_{\pi NN}/2M)^*$. Hence we write:

$$g^*_{\sigma NN} = g^{(q)*}_{\sigma NN} + \zeta_\sigma (g_{\pi NN}/2M)^{*2}, \tag{31}$$

$$g^*_{\rho NN} = g^{(q)*}_{\rho NN} + \zeta_g (g_{\pi NN}/2M)^{*2}, \tag{32}$$

$$\mu^*_{\rho NN} = \mu^{(q)*}_{\rho NN} + \zeta_f (g_{\pi NN}/2M)^{*2}. \tag{33}$$

The parameters, ζ_σ, ζ_g and ζ_f, represent the strengths of the pion cloud contributions. The quantities ζ_σ and ζ_g have the dimensions of mass squared , while ζ_f has the dimensions of mass. We choose reasonable values of $g_{\sigma NN}$, $g_{\rho NN}$ and $f_{\rho NN}/2M$ in free space and use the results of the TOY model in free space for $g^{(q)}_{\sigma NN}$, $g^{(q)}_{\rho NN}$, $\mu^{(q)}_{\rho NN}$ and $g_{\pi NN}/2M$ to fix the coefficients ζ_σ, ζ_g and ζ_f. These values and the relevant parameters are listed in either Table III or in the table caption.

We will see later in section V that the free space meson nucleon coupling constants used here are not exactly the same as in the one boson exchange potential used in the Dirac-Brueckner calculations. However, the differences are small and are not expected to affect the qualitative purpose of this paper.

As stated in the introduction a necessary input is the field-dependence of the exchanged meson masses. We use the simple model that the meson masses are linearly dependent on x [5], keeping the pion mass fixed. We assign the same x-dependence to both σ and ω masses.

$$m^* = m(1 - b_{meson}x) = m\phi(x). \tag{34}$$

The function $\phi(x)$ has been introduced so that the rest of the discussion is not specific to the simple linear dependence used in the numerical work. The coefficient b_{meson} is chosen to give $m^*/m = 0.92$ at normal nuclear density. Its values are listed in Table III.

TABLE III. List of parameters and coefficients. Both the PUREMASS and QUARTIC cases use $m_\sigma = 650$ MeV, $m_\omega = 783$ MeV, $m_\rho = 770$ MeV, $m_\pi = 140$ MeV, $g_{\sigma NN} = 8.0$, $g_{\rho NN} = 5.0$, $\mu_{\rho NN} = 2.45$ and $m^*/m = 0.92$ at normal nuclear density. The numbers in the table are all in pion mass units. The quantity $g_{\pi NN}/2M$ has the dimensions of length and is expressed in units of m_π^{-1}. The quantities $g_{\omega NN}$, $g^{(q)}_{\sigma NN}$, $g^{(q)}_{\rho NN}$ and b_{meson} are dimensionless. The quantity ζ_f has the dimensions of mass and is expressed in units of m_π. The quantities ζ_g and ζ_f have dimensions of mass squared and are expressed in units of m_π^2.

Quantity	PUREMASS	QUARTIC
$g_{\pi NN}/2M$	0.8349	0.9241
$g_{\omega NN}$	11.02	13.72
$g^{(q)}_{\sigma NN}$	2.389	4.919
ζ_σ	8.049	3.608
$g^{(q)}_{\rho NN}$	2.448	3.048
ζ_g	3.661	2.286
$\mu^{(q)}_{\rho NN}$	0.8349	0.9241
ζ_f	2.317	1.787
b_{meson}	0.4769	0.4622

IV. DENSITY DEPENDENCE

The first step in converting the information about field dependence of the coupling constants to density dependence is to obtain the density dependence of the fields, σ and ω , or equivalently, x and y, themselves. This is done through the following two basic self-consistency equations:

$$x = \frac{g^*_{\sigma NN}}{m^{*2}_\sigma}\xi\rho, \tag{35}$$

$$y = \frac{g^*_{\omega NN}}{m^{*2}_\omega}\xi\rho, \tag{36}$$

where the various quantities have been defined by Eqs. (15) and (34).

The factor ξ represents the reduction of the effective density of nucleon available to polarize a given interacting pair. The reduction is the combined effect of the Exclusion principle and short-range correlations among nucleons. An examination of the illustrative Goldstone graphs of Fig. 1 may be useful to understand the origin of this factor.

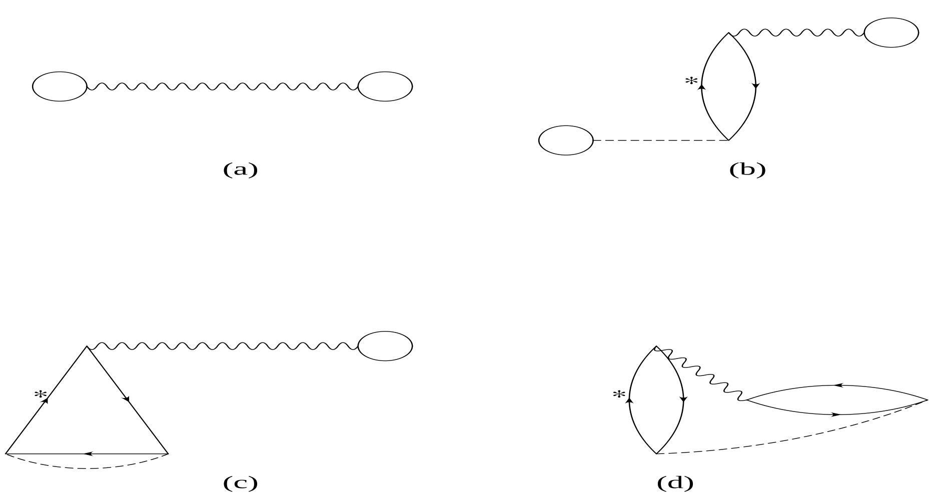

FIG. 1. The Fig. (a) represents interaction between two Fermi sea nucleons interacting via a G-matrix. In Fig. (b) a third nucleon excites one of the interacting nucleons into a resonance state, leaving a hole in the Fermi sea. The resonance propagator is shown as an upward line with a star. Fig. (c) is the result of exchange between the third nucleon and the left interacting nucleon. Recall the convention that a line starting and ending at exactly the same time represents a hole line. Fig. (d) represents the exchange between the third nucleon and the right interacting nucleon.

Besides the diagrams shown in the figure there are others obtained by interchanging the roles of the right and the left interacting nucleons. There are also diagrams where a fourth nucleon excites the interacting nucleons to a resonance state, generating two resonance-two hole states. The two resonance states interact and fall back into the hole states. All these are examples of single interaction of the neighboring nucleons with the interacting pair. These may be repeated and summed to generate field effects in a space containing the nucleon and $I = J = 1/2$ resonance states as degrees of freedom. If resonances were not included the procedure will generate contributions to the usual average field field of a many body theory where only the nucleon degree of freedom is counted. The mean field of CCM describes approximately the multidimensional average field described here.

Inclusion of all exchange diagrams enforces the Exclusion principle, which ensures that only two other nucleons can come close to the two interacting nucleons. The latter are mostly within a short distance. This consideration alone makes $\xi = 1/2$. Repulsive short range correlations make ξ even smaller. We expect that $0 \leq \xi \leq 0.5$. A full self-consistent many body treatment should determine this factor. But for the present preliminary study we take it to be an unknown parameter confined to the range $0 \leq \xi \leq 0.5$.

In Eq. (35) one should use the scalar density. This can be done only through a full many body self-consistent calculation. We simplify our work by using the vector density ρ. This simplifying strategy allows us to factor the problem of extracting the density dependence of the coupling constants from the rest of the many body problem. We estimate the error to be $\sim \langle \vec{p}^{\,2} \rangle / 4M^2 \simeq 0.05$. The strategy is probably justifiable in a preliminary study of the main question studied in this paper.

The Eqs. (35) and (36) may be rewritten as:

$$x = \{\Sigma^{(q)} F^{\sigma}(x,y) + \Sigma^{\pi} (F^{\pi}(x,y))^2\} \frac{1}{\phi^2(x)} \, \xi \frac{\rho}{\rho_0}, \tag{37}$$

where the combinations $\Sigma^{(q)}$ and Σ^{π} are

$$\Sigma^{(q)} = \frac{g^{(q)}_{\sigma NN}}{m_\sigma^2} \rho_0 \;\; \text{and} \;\; \Sigma^{\pi} = \zeta_\sigma (g_{\pi NN}/2M)^2 \frac{\rho_0}{m_\sigma^2}. \tag{38}$$

The quantity ρ_0 is the normal nuclear density. The quantities Σ^σ and Σ^π would be the σ fields produced by nuclear matter due to direct σ -quark coupling and due to σ -pion cloud coupling, respectively, if the coupling constants did not change with density. The x, y dependent quantities were defined by Eqs. (15) and (34).

For future convenience we introduce the dimensionless effective density $\bar{\rho}$:

$$\bar{\rho} = \xi \frac{\rho}{\rho_0}. \tag{39}$$

In a similar manner we write

$$y = \Omega \frac{F^\omega(x, y)}{\phi^2(x)} \, \bar{\rho}, \tag{40}$$

where

$$\Omega = \frac{g_{\omega NN}}{m_\omega^2} \, \rho_0, \tag{41}$$

would be the ω field produced by nuclear matter if the coupling constant did not change with density.

We solve the two coupled nonlinear Eqs. (37) and (40) for a range of $\bar{\rho}$ giving us the fields x and y as functions of $\bar{\rho}$. Using the results and Eqs. (15) we obtain numerical values of the five coupling constants, $g^*_{\sigma NN}$, $g^*_{\omega NN}$, $(g_{\pi NN}/2M)^*$, $g^*_{\rho NN}$, $\mu^*_{\rho NN}$ and m^*/m as functions of $\bar{\rho}$.

We remind the reader that we do not use the absolute coupling constants from the CCM calculation in the Dirac-Brueckner treatment of nuclear matter. We use only the results for the density dependences of the ratios such as $g^*_{\sigma NN}/g_{\sigma NN}$.

These ratios are plotted as functions of $\bar{\rho} = \xi\rho/\rho_0$ in Fig. 2. To make it convenient to use these results in a relativisitic Dirac-Brueckner-Hartree-Fock calculation we represent the density dependences in the form of a $[4, 5]$ rational function:

$$\frac{g^*}{g} = 1 + \frac{\sum_{\ell=1,4} \alpha^g_\ell \bar{\rho}^\ell}{1 + \sum_{n=1,5} \beta^g_n \bar{\rho}^n}, \tag{42}$$

$$\frac{m^*}{m} = 1 + \frac{\sum_{\ell=1,4} \alpha^m_\ell \bar{\rho}^\ell}{1 + \sum_{n=1,5} \beta^m_n \bar{\rho}^n}. \tag{43}$$

The coefficients α_ℓ and β_n for the five coupling constants and the meson mass ratio are obtained by least square fitting and are listed in Tables IV and V.

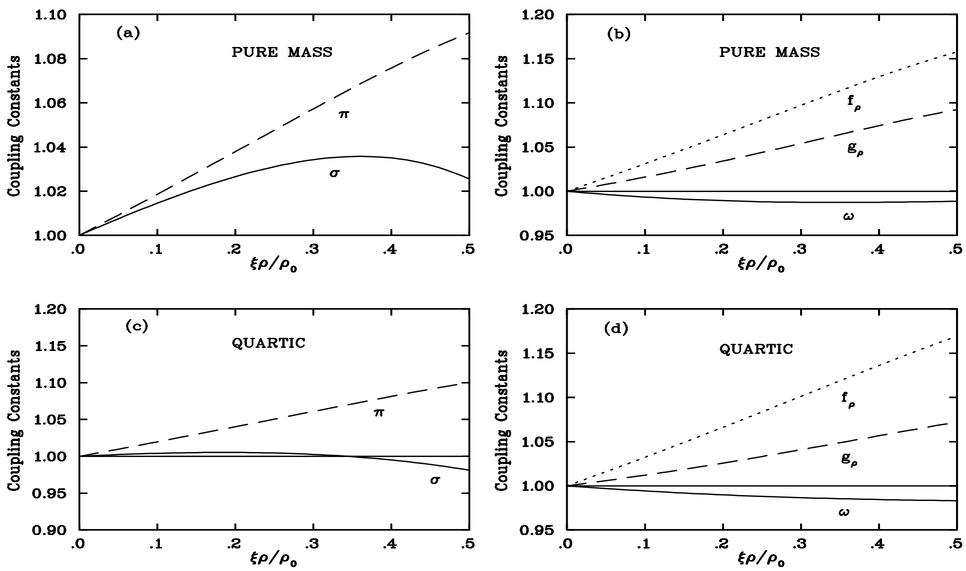

FIG. 2. Plots of $g^*_{\sigma NN}/g_{\sigma NN}$ labelled with σ , $(g_{\pi NN}/2M)^*/(g_{\pi NN}/2M)$ labelled with π, m^*/m, $g^*_{\omega NN}/g_{\omega NN}$ labelled with ω, $g^*_{\rho NN}/g_{\rho NN}$ labelled with g_ρ and $\mu^*_{\rho NN}/(\mu_{\rho NN})$ labelled with f_ρ as functions of $\xi\rho_N/\rho_0$. Results for both the PUREMASS and the QUARTIC forms for $U(\chi)$ are presented. All relevant parameters are listed in Table III in section III.

TABLE IV. Coefficients of rational functions defined in Eqs. (42) and (43) obtained with PURE MASS $U(\chi)$. Columns 2 through 6 are for the ratios of coupling constants, while the last column is for m^*/m. Rows labelled 1 through 4 are the numerator polynomial and the remaining rows are for the denominator polynomial. All parameters relevant to the calculations are listed in Table III in section III.

	$\frac{g^*_{\sigma NN}}{g_{\sigma NN}}$	$\frac{g^*_{\omega NN}}{g_{\omega NN}}$	$\frac{(g_{\pi NN}/2M)^*}{g_{\pi NN}/2M}$	$\frac{g^*_{\rho NN}}{g_{\rho NN}}$	$\frac{\mu^*_{\rho NN}}{\mu_{\rho NN}}$	m^*/m
1	0.1553	-0.0779	0.1817	0.1491	0.3015	-0.0877
2	-0.4452	0.2303	-0.4327	-0.2797	-0.7174	0.1535
3	0.4843	-0.2198	0.4802	0.2101	0.7951	-0.1481
4	-0.2853	0.0484	-0.2026	-0.0700	-0.3352	0.0295
5	-2.3792	-1.3216	-2.6652	-2.6050	-2.7329	-2.1338
6	2.8749	0.3065	3.6722	3.1574	3.8297	2.6547
7	-1.7181	1.0527	-2.6344	-1.8743	-2.8004	-1.6752
8	0.6688	-0.6230	1.2151	0.5764	1.2733	0.9508
9	-0.0517	0.1177	-0.1483	-0.0486	-0.1415	-0.1372

TABLE V. Coefficients of rational functions defined in Eqs. (42,(43) obtained with QUARTIC $U(\chi)$. Columns 2 through 6 are for the ratios of coupling constants, while the last column is for m^*/m. Rows labelled 1 through 4 are the numerator polynomial and the remaining rows are for the denominator polynomial. All parameters relevant to the calculations are listed in Table III in section III

	$\frac{g^*_{\sigma NN}}{g_{\sigma NN}}$	$\frac{g^*_{\omega NN}}{g_{\omega NN}}$	$\frac{(g_{\pi NN}/2M)^*}{g_{\pi NN}/2M}$	$\frac{g^*_{\rho NN}}{g_{\rho NN}}$	$\frac{\mu^*_{\rho NN}}{\mu_{\rho NN}}$	m^*/m
1	0.0522	-0.0653	0.1928	0.1117	0.3129	-0.0852
2	-0.2078	0.1460	-0.3308	-0.2173	-0.5382	0.1093
3	0.2027	-0.1240	0.2994	0.1304	0.4855	-0.0842
4	-0.1174	0.0297	-0.1087	-0.0230	-0.1756	0.0140
5	-1.9719	-1.0893	-1.9685	-2.7149	-2.0459	-1.5207
6	2.0516	0.2868	2.2234	3.1836	2.3486	1.4465
7	-1.0666	0.5555	-1.2896	-1.9596	-1.3931	-0.6018
8	0.3575	-0.2999	0.5296	0.5796	0.5573	0.2730
9	-0.0292	0.0451	-0.0640	-0.0663	-0.0601	-0.0200

V. DIRAC-BRUECKNER ANALYSIS

Modifications of the meson masses and meson-nucleon coupling constants by the presence of the nuclear medium as discussed in the previous sections will, in general, lead to changes in the saturation properties of nuclear matter. This problem can be studied within the relativistic Dirac-Brueckner approach. To carry out such a study we follow the approach described in Ref. [8]. As dynamical input we use the relativistic one-boson-exchange model of Ref. [9]. The NN T-matrix satisfies a Bethe-Salpeter equation, which is formally given by

$$T = V + V S_2 T, \tag{44}$$

where V is the NN interaction and S_2 is the free 2-nucleon Green function. In this study we use in particular the quasi-potential version of it. The interaction V is assumed to be given by the exchange of π, σ, ρ, ω, δ and η mesons. In particular, in the model a derivative pion and η meson coupling has been assumed. For our study we choose the coupling parameters of interaction A, which gives a reasonable fit to the NN scattering phase shifts [32]. To regulate the behaviour at high momenta, a monopole form factor $\Lambda^2/(k^2 - \Lambda^2)$ has been introduced at each meson-nucleon vertex. A cutoff mass of $\Lambda = 1150$ MeV has been taken. In Table VI the meson parameters of the model A are listed.

TABLE VI. Meson-nucleon coupling constants g and meson masses m of the one-boson-exchange model A

	π	σ	ω	ρ^v	ρ^t/ρ^v	δ	η
$g^2/4\pi$	14.2	7.6	11.0	0.43	6.8	0.75	3.09
m (MeV)	139	570	783	763	763	960	548

According to the CCM model the free space meson parameters will be modified in the presence of the nuclear medium. Within the one-boson-exchange model the medium modifications found in the previous section can be simply implemented by replacing for each meson ϕ the coupling constant $g_{\phi NN}$ and the mass m in the one-boson-exchange interaction V by the corresponding g^*_{mNN} and m^* given by Eqs. (42) and (43). Moreover, the Pauli-blocking due to the medium has to be accounted for in Eq. (44), leading to the Bethe-Brueckner-Goldstone equations for the G-matrix. This is done by replacing the Green function in Eq. (44)

$$S_2 \to S(p_1)S(p_2)Q \tag{45}$$

where $S(p_n)$ are the medium-modified nucleon propagators and Q the Pauli-blocking operator, which projects out in the intermediate states nucleon momenta inside the Fermi sphere. Introducing the baryonic current $B = \rho\, u$, with ρ being the density and u the unit vector $u = (1, \vec{0})$ in the nuclear-matter frame, due to Lorentz covariance we may write the nucleon self-energy contribution in terms of the 3 invariants Σ^α

$$\Sigma(p) = \Sigma^s - \Sigma^0 \gamma.u - \Sigma^v \gamma.p_\perp, \tag{46}$$

where $p_\perp = p - (p.u)u$. The medium-modified nucleon propagator can be approximated near the dressed nucleon pole as

$$S(p) = [E^* \gamma_0 - \vec{p}\vec{\gamma} - M^*]^{-1} \tag{47}$$

with

$$\begin{aligned} M^* &= (M + \Sigma^s)/(1 + \Sigma^v) \qquad (48) \\ E^* &= (p_0 + \Sigma^0)/(1 + \Sigma^v) \end{aligned}$$

For the Pauli-blocking operator Q an angular averaged approximation has been made. The resulting relativisitic G-matrix equations

$$G = V + V S(q_1)S(q_2)QG \tag{49}$$

with q_n the momenta of the nucleons in the intermediate states, are solved in the NN CM frame after partial wave decomposition. This is done within a quasi-potential equation description using the helicity basis of positive and negative energy spinor states corresponding to mass M^*.

Relativistic nuclear matter calculations require the knowledge of the transformation properties of the G-matrix under Lorentz transformations. After having determined the relativistic G-matrix in the 2-particle CM system at a fixed matter density, we use the IA2 representation [8] to obtain it in the nuclear matter frame. This representation gives the complete covariant form of the G-matrix in an unambiguous way. To reconstruct it all matrix elements of the amplitude in the full Dirac space are needed. From this IA2 representation, neglecting the vacuum fluctuation terms, the self-energy can be readily determined. Taking nucleon 2 to be one of the valence nucleons in the Fermi sphere with Fermi momentum p_f, we have

$$\Sigma(p) = -Tr_2 \int_{|p_2|<p_f} \frac{d\vec{p}_2}{(2\pi)^3 2E_2^*} < p, p_2|G|p, p_2 >, \tag{50}$$

where nucleon 2 is in a positive energy state with momentum p_2 and Tr_2 designates the summation over the spin index of this nucleon and $E_2^* = \sqrt{p_2^2 + M^{*2}}$. Hence the self-energy contribution is essentially obtained by taking the diagonal matrix-elements of the G-matrix with one of the nucleons belonging to the filled Fermi sphere while the other one carries a momentum p.

Since the G-matrix depends implicitly on the Σ^α's through Eq. (47) the calculations have to be carried out in a self-consistent way. In doing so, the self-energy can be determined for a given matter density ρ. In particular we have assumed that Σ does not vary significantly within the Fermi sphere, so that its value can be taken at the Fermi momentum p_f. Some studies [6] of the momentum dependence have been made, indicating that the variations in Σ are of the order of 10 %. Hence this is a reasonable approximation.

The partial wave decomposed integral equations for the G-matrix have been solved using Gaussian quadratures. Typically 24 Gaussian points are sufficient to get an accuracy of a

few percent. Moreover, 6 Gaussian points for each integration variable in Eq. (50) have been used to determine the self-energies. The self-consistent solutions can be found in an iterative way. Adopting a starting value for the Σ's in Eq. (48), Eq. (49) is solved to yield through Eq. (50) new values for the Σ's. Varying subsequently the Σ's we then can determine the solutions of Eqs. (49-50) such that the Σ's are the same. Once the selfconsistent self-energy solutions have been found the binding energy of the ground state can readily be calculated from the energy-momentum tensor as a function of density. For more details of Dirac-Brueckner calculations we refer to Refs [6] and [8].

For the various relativistic one boson interactions we find, in general, that the system exhibits saturation. As found in Ref [8], an exception to this is when one assumes that only the meson-masses drop as $m^*/m = M^*/M$ [5]. It is interesting to note that in the case we also allow for medium-modification of the meson coupling constants as is predicted in our study of the chiral confining model, saturation does occur, that is we find that the binding energy of the ground state as a function of the matter density ρ has a minimum. In the next section we discuss our results for the saturation properties of nuclear matter as predicted for the medium modifications of the coupling constants as found in our CCM model.

VI. RESULTS AND DISCUSSIONS

A. Main Results

As we have discussed earlier, the coupling constants are dependent not directly on the density ρ of nuclear matter, but on the effective density, $\xi\rho$, seen by an interacting pair in its immediate neghborhood. The Pauli Exclusion principle ensures that $\xi < 1/2$. There is certainly further reduction of ξ due to strong repulsion at short distances. Thus the range of ξ of interest is $0 \leq \xi \leq 0.5$.

We pick a value of ξ and proceed to calculate E/A and ρ at saturation using the Dirac-Brueckner approach described in Section V. In the calculations reported here we have

used the one boson exchange interaction A specified in Ref. [8]. During each loop of self-consistency iteration we use coupling constants and meson masses appropriate for $\bar{\rho}_N = \xi\rho/\rho_0$, defined earlier by Eq. (39), ρ being the nuclear density at the particular stage of the iteration. Specifically, we use Eqs. (42) and (43) with the coefficients of the rational functions listed in Tables IV and V. Finally, self-consistency is achieved and we have a pair of numbers, E/A and density ρ at saturation for the given value of ξ. Figures 3 and 4 show these results for the PUREMASS and the QUARTIC cases, respectively.

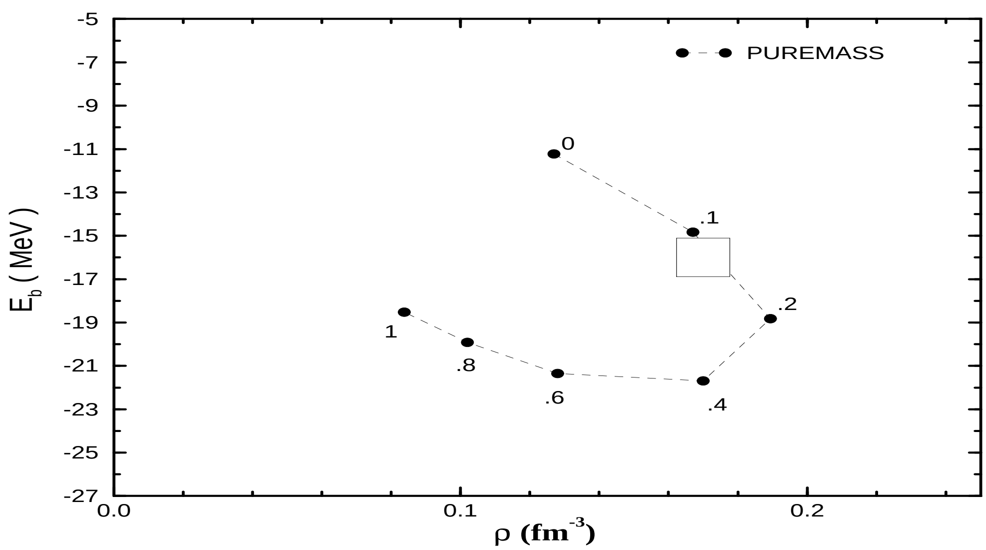

FIG. 3. Results of E/A and ρ at saturation for the PUREMASS case for a series of values of $0 \leq \xi \leq 1.0$, shown with solid circles. The square box represents the empirical nuclear matter density and binding energy. The values of ξ appear next to the solid circles. The line is for guiding the eye only. The details of calculations are given in section V. Only the range $0 \leq \xi \leq 0.5$ is of physical interest.

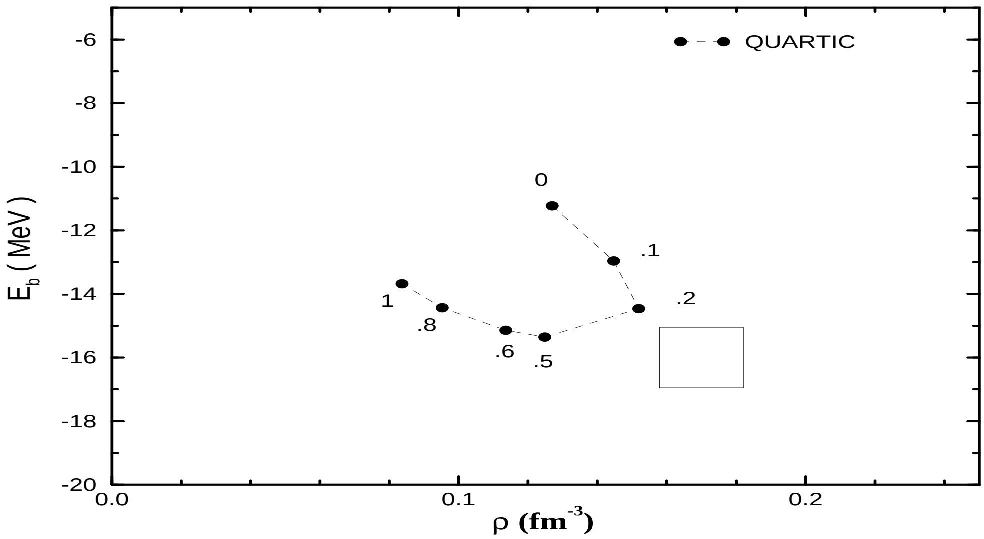

FIG. 4. Results of E/A and ρ at saturation for the QUARTIC case for a series of values of $0 \leq \xi \leq 1.0$, shown with solid circles. The square box represents the empirical nuclear matter density and binding energy. The values of ξ appear next to the solid circles. The dashed line is for guiding the eye only. The details of calculations are given in section V. Only the range $0 \leq \xi \leq 0.5$ is of physical interest.

The value corresponding to $\xi = 0$ represent the Amorim and Tjon [8] result. From the figures we see that dependence of the nuclear saturation properties on ξ is just of the right size to be of interest in the present study. The E/A vs. ρ curve for the PUREMASS case passes through the square representing experimental data for ξ slightly greater than 0.1. For the QUARTIC case the E/A vs. ρ curve comes close to the square, but does not enter it. The closest approach occurs for ξ a little larger than 0.2. The results for the unphysical range, $\frac{1}{2} < \xi \leq 1$ are included in the figures to exhibit the curious looping effect.

B. Discussions

In order to study the origin of the looping effect we have systematically switched off the density dependences of the various coupling constants and meson masses. In so doing, we find that the density dependence of $g^*_{\sigma NN}$ is the main source of the looping behavior of the saturation values of E/A and ρ. Qualitatively such a behaviour can be understood with a simple model where the essential ingredient of the density dependence in the coupling constant $g^*_{\sigma NN}$ is built in as a small modification of the saturation curve.

Assuming that all meson masses and all coupling constants, except $g^*_{\sigma NN}$ are independent of ρ_N, the density dependence of E/A can be modelled in the following manner:

$$E/A = (E/A)_0 + \frac{1}{2}K_0(y-1)^2 - \frac{3}{4}\rho\frac{g_{\sigma NN}(0)^2}{m_\sigma^2}\left[(g_{\sigma NN}(\rho)/g_{\sigma NN}(0))^2 - 1\right], \tag{51}$$

where, for convenience, we have introduced the quantity

$$y = \rho/\rho_0, \tag{52}$$

The first two terms describe the density dependence of E/A near the saturation point when none of the coupling constants and meson masses is density dependent. The quantity K_0 is the nuclear incompressibility and $(E/A)_0$ is the minimum value of E/A in this situation. The last term contains the effect of the density dependence of $g^*_{\sigma NN}$, calculated perturbatively, using, in addition, the fact that $\overline{(\vec{p}_1 - \vec{p}_2)^2} \ll m_\sigma^2$ *i.e.,* the average squared relative momentum of two Fermi sea nucleons is much smaller than the square of the σ meson mass.

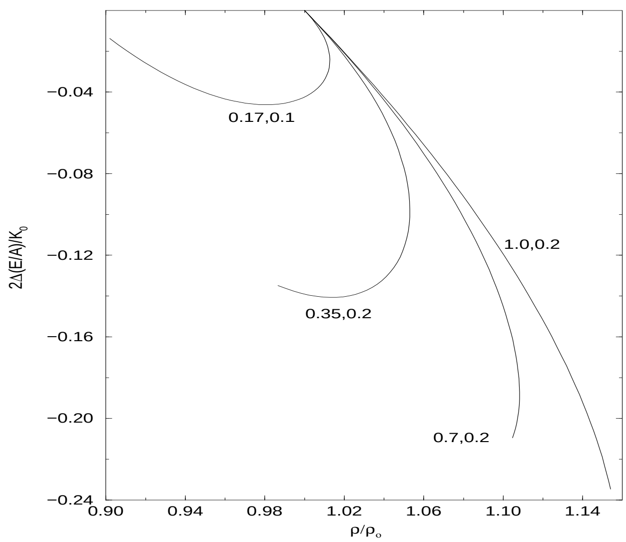

FIG. 5. Plots of $2\Delta(E/A)/(280MeV)$ vs. the saturation density ρ/ρ_0 for $K_0 = 280$MeV and $(y_1, \eta) = (0.17, 0.1)$, $(0.35, 0.2)$, $(0.7, 0.2)$, and $(1.0, 0.2)$.

An inspection of the two graph in the left column of Fig. 2 shows that the quadratic form:

$$g_{\sigma NN}(\rho)/g_{\sigma NN}(0) = 1 + \eta\xi y(1 - \frac{\xi y}{2y_1}), \tag{53}$$

can describe these graphs quite well. The ratio $g_{\sigma NN}(\rho)/g_{\sigma NN}(0)$ peaks at $\xi y = y_1$ and the peak value is $1 + \eta y_1/2$. The values of the various parameters used in the illustration are $K = 280MeV$, $g_{\sigma NN} = 8.0$, $m_\sigma = 630MeV$. We consider four sets of y_1 and η, namely, $(y_1, \eta) = (0.17, 0.1)$, $(0.35, 0.2)$, $(0.7, 0.2)$, and $(1.0, 0.2)$. The first two describe the ratios $g_{\sigma NN}(\rho)/g_{\sigma NN}(0)$ for the QUARTIC and the PUREMASS cases, respectively, in Fig. 2. The last two are used to illustrate the origin of looping.

It is convenient to recast Eq. (51) into the form:

$$2[E/A - (E/A)_0]/K_0 = (y-1)^2 - y\frac{3\rho_0}{4K_0}\frac{g_{\sigma NN}(0)^2}{m_\sigma^2}\left[(g_{\sigma NN}(\rho)/g_{\sigma NN}(0))^2 - 1\right]. \tag{54}$$

Because of the presence of the third term in Eq. (51) the saturation density will change. For a given value of ξ this can be readily determined by minimizing the expression for E/A

with respect to $y = \rho/\rho_0$. The resulting plots of the change in the binding energy at the new saturation density vs. ρ/ρ_0 are shown in Fig. 5 for the four sets of where ξ has been varied in the region $0. \leq \xi \leq 0.5$.

Starting from zero density, as $g_{\sigma NN}(\rho)$ increases with $\bar{\rho}$ the quantity E/A becomes more negative. The saturation properties reflect this through a combination of increasing saturation density, ρ, and decreasing E/A. Eventually, decrease of $g_{\sigma NN}(\rho)$ beyond its peak with increasing $\bar{\rho}$ and the effect of incompressibilty take over and the saturation density stops increasing. If this occurs in the range $0 \leq \xi \leq 0.5$, we see looping. If the peak of $g_{\sigma NN}(\rho)$ occurs at a higher density, *i.e.*, for a higher y_1, the saturation density keeps on increasing and we see no looping for $\xi \leq 0.5$. In the present example the critical value of y_1 for $\eta = 0.2$ appears to be slightly higher than 0.7. Thus the occurence of a peak of $g_{\sigma NN}(\rho)$ at not too high a value of the effective density is needed for looping to occur for $\xi \leq 0.5$. The value of y_1 is lower for the QUARTIC case than for the PUREMASS case. This explains why the loop turns for the former case at a density lower than that for the latter.

C. Concluding Remarks

Due to the internal structure of the nucleon, we should, in general, expect that the effective meson nucleon parameters may change in nuclear medium. Using a chiral confining model we have studied the density dependent changes of the meson-nucleon coupling parameters. We have also used a simple ansatz for the density dependence of σ and ω masses. Using the framework of a Dirac-Brueckner analysis we have found that their effect on the saturation properties of nuclear matter can be significant. Due to the density dependence of the $g^*_{\sigma NN}$ as predicted by the chiral confining model we have found, in particular, a looping behaviour in the nuclear matter binding energy at saturation density. The looping has essentially also been verified in a simple model, where it is mainly caused by the presence of a peak in the density dependence of the medium modified σN coupling constant at a low density. We should stress that the effect of the density dependence of the other quantities are

not negligible. However, a qualitative understanding of the looping effect can be obtained by paying attention to the density dependence of $g_{\sigma NN}$ alone.

It appears that the density dependence of the coupling constants and the meson masses produce effects which are small but interesting. In particular the small effect tends to improve the results for nuclear matter. From the present study we see that the relationship between binding energy and saturation density may not be as universal as found in nonrelativistic studies and that more model dependence is exhibited once medium modifications of the basic nuclear interactions are considered. We hope that these preliminary results will encourage more detailed investigation of this particular variety of density dependence of the NN interaction.

VII. ACKNOWLEDGEMENTS

This work was supported by DOE Grant DOE-FG02-93ER-40762. Bulk of the writing and part of the work by one of us (MKB) were done while visiting the Institute of Kernphysik, Forschungszentrum Jülich. He thanks his colleagues there for their hospitality. His visit was made possible by a Alexander von Humboldt Foundation award for senior U.S. physicist and he thanks the Foundation for it.

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